INDIAN SCHOOL AL WADI AL KABIR

Assessment -I

Class: XII **MATHEMATICS (Code: 041)** Max Marks: 80
Date:29/09/2022 Time: 3 hours

General Instructions:

This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of Assessment (4 marks each) with sub parts.

SECTION – A

1 The relation R in the set of real numbers defined as $x - y + \sqrt{3}\ is\ irrational, then\ R\ is$ 1

a) reflexive b) transitive c) symmetric d) None of these

2 Let R be a relation defined as R = {(x, y):$x^2 + y^2 \leq 4, x, y \in Z\}$, then domain of Z 1

a) {0, 1, 2} b) {1,2}
c) {-2, -1, 0, 1, 2} d) {1, 2, 3, 4}

3 $If\ A = \begin{bmatrix} 3 & 4 \\ 1 & 3 \end{bmatrix}\ and\ B = \begin{bmatrix} 2 & -2 \\ 3 & -1 \end{bmatrix}\ then\ (A+B)^{-1} is$ 1

$a. \begin{bmatrix} -1 & 1 \\ 1 & -\frac{5}{2} \end{bmatrix}$ $b. Does\ not\ exist$ $c. \begin{bmatrix} 1 & 1 \\ 1 & \frac{5}{2} \end{bmatrix}$ $d. \begin{bmatrix} 1 & -1 \\ -2 & \frac{5}{2} \end{bmatrix}$

4 If A + B = $\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$ and A – B = $\begin{bmatrix}-1 & 0\\1 & -1\end{bmatrix}$, then A is 1

a) $\begin{bmatrix}0 & 0\\1 & 1\end{bmatrix}$ b) $\begin{bmatrix}0 & 0\\1 & 0\end{bmatrix}$ c) $\begin{bmatrix}\frac{1}{2} & 0\\1 & \frac{1}{2}\end{bmatrix}$ d) $\begin{bmatrix}\frac{1}{2} & \frac{1}{2}\\1 & 1\end{bmatrix}$

5 $cos^{-1}\left[cos\left(\frac{9\pi}{4}\right)\right]$ 1

a) $-\frac{\pi}{4}$ b) $\frac{\pi}{4}$ c) $-\frac{\pi}{8}$ d) $\frac{\pi}{8}$

6 The value of k for which the following function is continuous at x = 3 is 1

f (x) = $\begin{cases}\frac{kcosx}{\pi-2x}, & x \neq \frac{\pi}{2}\\ k, & x = \frac{\pi}{2}\end{cases}$

a) -6 b) -3 c) 0 d) 6

7 If y = Ae^{7x} + Be^{-7x}, then $\frac{d^2y}{dx^2}$ is equal to 1

a) 7y b) -7y c) 49y d) -49y

8 The value of $\tan^{-1}\sqrt{3}$ + $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ is 1

a) $-\frac{\pi}{3}$ b) $\frac{\pi}{3}$ c) π d) $-\pi$

9 If A = $\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$ and B = $\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$ then AB is 1

a) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ b) $\begin{bmatrix}1 & 0\\3 & 1\end{bmatrix}$ c) $\begin{bmatrix}1 & 0\\0 & 3\end{bmatrix}$ d) $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$

10 If a matrix is both symmetric and skew symmetric, then A is necessarily 1

a. a diagonal matrix b. a zero square matrix c. a square matrix d.an identity matrix

11 The maximum value of $|1+\sin 2x|\ is$ 1

a) 1 b) 2 c) 3 d) 0

12 The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15) and (0, 20). Let Z = px + qy, where p, q>0. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is 1

a) $p=q$ $b)\ p=2q$ $c)\ p=\ 3q$ $d)\ 3p=\ q$

13 If C_{ij} is the cofactor of P_{ij}, where $P=\begin{bmatrix}2 & 0 & 1\\ -1 & 0 & 4\\ 3 & 2 & 1\end{bmatrix}, then\ C_{13}.C_{23}$ is 1

a) 8 b) -12 c) 12 d) 0

14 If set A has 5 elements and B has 6 elements then the number of one to one mapping from A to B is 1

a) 120 b) 30 c) 540 d) 720

15 $When\ x>0, \int \frac{dx}{xlogx}$ is equal to 1

$a)\ log(logx)+\ c$ $b)\ log\ (xlogx)\ +\ c$ $c)\ 2log\ x\ +\ c$ $d)\ log\ (2logx)\ +\ c$

16 $\int \frac{1}{sin^2xcos^2x}dx$ is equal to

$a)\ sinx - \ cosx\ + c$ $b)\ cosecx\ + cotx\ +\ c$ $c)\ tanx\ - cotx + \ c$ $d)\ sin2x\ +\ c$

17 If $\begin{vmatrix}x & 6\\ 4 & x\end{vmatrix} = \begin{vmatrix}2x & 5\\ 5 & x\end{vmatrix}$, then the value of x is 1

a) $\pm\sqrt{5}$ b) ± 1 c) ± 7 d) $\pm 2\sqrt{6}$

18 If the radius of a circle changes at the rate of 3cm/s, then the rate of change of area when the radius is 5cm. 1

a) $30\pi cm^2/s$ b) $15\pi cm^2/s$ c) $\pi cm^2/s$ d) $6\pi cm^2/s$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

19 Assertion (A): The value of k for which the function f(x) = $\begin{cases} \frac{e^{3x}-e^{-3x}}{x} ; x \neq 0 \\ k ; x = 0 \end{cases}$ is continuous at x = 0 when k = 6 1

Reason (R): A function f(x) is continuous at a point x = a of its domain if $\lim_{x \to a} f(x) = f(a)$

20 Assertion (A): If A= $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & 1 & k \end{pmatrix}$ is singular, then $k = \frac{1}{2}$ 1

Reason (R): For any square matrix A of order n, $|A| = 0$

SECTION - B

21 Evaluate $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$ 2

22 If A is a square matrix such that A^2 = A, then find the simplified value of $(A-I)^3 + (A+I)^3$. 2

OR

If A = $\begin{bmatrix} 2 & 4 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}$, evaluate $(AB)'$.

23 Express $A = \begin{pmatrix} 4 & -2 \\ 3 & 5 \end{pmatrix}$ as a sum of a symmetric and a skew symmetric matrix. 2

24 Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast height of the sand cone is increasing when the height is 4cm? 2

25 Find $\int \frac{x}{x^2-1}\,dx$ OR Find $\int \frac{\sin x}{\sin(x-a)}\,dx$ 2

SECTION - C

26 If $x = a\left(\log\tan\frac{\theta}{2} + \cos\theta\right), y = a\sin\theta$ then find $\frac{dy}{dx}$ 3

OR

If $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$, then prove $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$

27 If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ then prove $A^2 - 3A - 7I = 0$. Hence find A^{-1}. 3

28 If $y = 3\cos(\log x) + 4\sin(\log x)$, prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ 3

OR

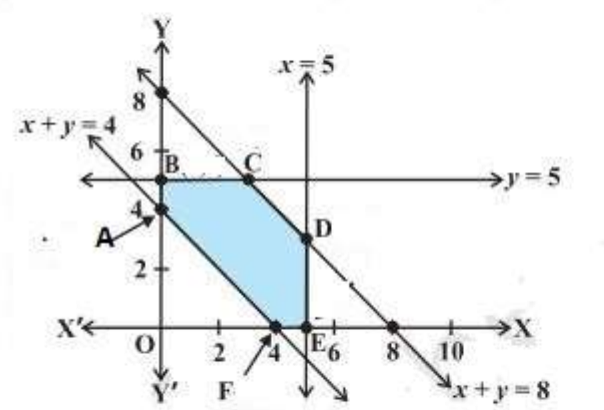
Find the intervals in which the function $f(x) = 4x^3 - 6x^2 - 72x + 30$ is strictly increasing or decreasing.

29 Evaluate $\int \frac{1}{1+\cot x}\,dx$ 3

OR

Evaluate $\int \frac{\sqrt{\tan x}}{\sin x \cos x}\,dx$

30 In the given graph, the feasible region for an LPP is shaded. 3



a. What are the constrains for the feasible region ABCDEF?
b. Evaluate $Z = 10(x - 7y + 190)$ if Z is minimum at C.

31 Let $A = R - \{3\}$, $B = R - \{1\}$. Let $f: A \to B$ be definded by $f(x) = \frac{x-2}{x-3}, x \in A$. Prove that f is bijective 3

SECTION - D

32 Show that the relation R defined on set A = $\{0, 1, 2, 3, \dots .12\}$ 5

R = $\{(a, b): |a - b|$ is divisible by $4; a, b \in A\}$ is an equivalence relation

OR

Prove that the relation R on the set N X N defined by (a, b) R (c, d), if $ad = bc$, for all (a, b), (c, d) ϵ N X N is an equivalence relation.

33 $If\ A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\ and\ B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$, find AB 5

Hence use the product to solve the system of equations

$x - y + 2z = 1$
$2y - 3z = 1$
$3x - 2y + 4z = 2$

OR

Solve the system of equations:

$x - 2y + z = 0$
$2x - y - z = 3$
$2y + z = 5$

34 Solve the following Linear Programming Problems graphically: 5

$Maximize\ Z = 3x + 5y$

$subject\ to: x + 3y \leq 60, x + y \geq 10, x \leq y,\ x, y \geq 0.$

35 An open topped box is to be constructed by removing equal squares from each corner of 5

$18cm \times 18cm$ square sheet of aluminum and folding up the sides. Find the volume of the largest box.

OR

Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle 30^0 is one third that of the cone and the greatest volume of cylinder is $\frac{4}{81}\pi h^3$.

SECTION - E

This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (a), (b) and (c) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36 Hari visited an exhibition along with his family. The exhibition had a huge swing. Hari found that the swing traced the path of a Parabola as given by $f(x) = x^2 + 1$ 4

Answer the following questions based on the above informations:

a. Given: $f(x) = x^2 + 1,\ f: R \to R$. Show that f is not an injective function.

b. If $f(x) = x^2 + 1, f: \{1, 2, 3, 4, \ldots\} \to X$, then write the range X .

c. Evaluate the minimum and maximum values (if any) of $f(x) = x^2 + 1, x \in R$ **OR** Find the intervals in which the function $f(x) = f(x) = x^2 + 1, x \in R$ is strictly increasing or decreasing.

37 Three schools A, B and C decided to organize a fair for collecting money for helping the flood victims. They sold handmade fans, mats and plates from recycled material at a cost of ₹ 50, ₹ 100 and ₹ 40 each respectively. The numbers of articles sold are given as 4

School ↓	Handmade fans	Mats	Plates
A	40	40	25
B	50	40	50
C	30	20	40

a. Express the given data in matrix form to find the amount collected by each school.

b. If A is a 3×3 a matrix and B is a 3×1 matrix what is the order of AB?

c. What is the total money collected by the school A? **OR** What is the total money collected by selling handmade fans by all the three schools?

38 A manufacturer making toys can sell x items at a price of ₹ $\left(5-\frac{x}{100}\right)$ each. 4

Cost price of one item is ₹$\left(\frac{1}{5}+\frac{500}{x}\right)$.

Base on the above answer the following:

a. Write selling price S(x), cost price C(x) and the profit function P(x) where x is the number of items manufactured and sold.
b. Find the number of items he should sell to earn maximum profit.

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